Investigating the Effect of Internal Pressure and Thickness of Thick-Walled Cylindrical Vessels on the Ratcheting Strains under Compressive Cycling Loading Using the Quasi-Creep Method

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Abstract

Thick-walled vessels have many applications in military, chemical, and aerospace industries and also in nuclear facilities. Increasing the internal pressure inside these vessels can take some of the layers of the vessel into the plastic zone. If this happens several times, we will see the accumulation of plastic strains called ratcheting. This paper assumes that the thick-walled vessel is subjected to a cyclic internal pressure between zero and a maximum value. In order to analyze this phenomenon, first, we present the quasi-creep method, and then we validate this method using the finite element Abaqus Software based on the combined hardening model. Then we employ this method to evaluate the effect of internal pressure and thickness of the vessel on the amount of ratcheting strains in different cycles. In the end, the results of this research and the accuracy and speed of the quasi-creep method are stated.

1. Introduction

Due to the importance of thick-walled vessels in different industries, it is imperative to pay special attention to different aspects of designing these types of vessels. One of the issues that have received less attention is the ratcheting phenomenon. When a material under asymmetric cycling loading is subjected to stresses that exceed the yield limit, plastic strains begin to accumulate, and this phenomenon is called ratcheting [1]. Given that many engineering structures are subjected to cycling loading, neglecting this issue can lead to disastrous failures. Understanding the ratcheting phenomenon is also significant in predicting fatigue life [2].

Ratcheting strain depends on many factors, the most important of which is stress average and amplitude and loading history [3]. Zhang & Chen [4] examined the ratcheting phenomenon in a rigid cylinder with constant axial stress and cycling shear stress and emphasized the loading history. They showed that ratcheting strains under this loading condition will increase axial stress and shear stress range. Several models have been presented so far to predict the ratcheting behaviour of the pipe under cyclic bending loading with a uniform internal pressure [5]. This study has shown that the presented models cannot predict the ratcheting strains very accurately. Among the reviewed methods, the combined non-linear hardening method introduced by Chaboche [6] predicts the ratcheting strains to a relatively acceptable level. Chen and Jiao [7] also worked on the laws of the combined hardening model to predict ratcheting strains in multiaxial loading and presented a modified model. In another study, the Chaboche model is named one of the complete models in analyzing the ratcheting behaviour of materials, among other models such as Uno-Wang, Armstrong-Frederick, and Prager. Therefore, this study exploits the combined non-linear hardening model, Chaboche, to model the ratcheting phenomenon in a thick-walled cylindrical vessel in Abaqus Software [8, 9].

Obuli et al. [10] performed a finite element analysis on the ratcheting behaviour in straight pipes under different loadings of constant internal pressure and cyclic bending loading. Gao [11] also examined the ratcheting behaviour of low-carbon steel pipes under compressive and cycling bending loading and modelled it using finite element analysis. In
the studies conducted so far, the type of cycling compressive loading in the thick-walled vessel has not been investigated, but it will be discussed in this research.

Previous studies have proven many similarities between the ratcheting behaviour and the creep behaviour in materials [12-15]. Additionally, some other researches have shown that ratcheting can be divided into three creep-like zones: (1) the first zone, where the ratcheting strain rate decreases (2) the second zone where the ratcheting rate stays almost the same (3) the third zone where we see the rapid growth of the ratcheting rate [16]. Based on the similarity seen between these two phenomena, this study presents a quasi-creep method that models the ratcheting phenomenon and uses the variable material properties method.

The variable material properties method presented by Jahed [17] helped him in analyzing the disk plastic and the thick-walled vessel. He also analyzed the creep in the rotating disk and the thick-walled vessel using the same variable material properties method [18].

In the quasi-creep method, it is assumed that the internal pressure of the vessel at each cycle varies between zero and the maximum. Finite element analysis is performed in Abaqus Software based on the Chaboche model (i.e. combined non-linear hardening) to validate the quasi-creep method. Finally, the effect of maximum internal vessel pressure and vessel thickness on ratcheting strains is evaluated using the quasi-creep method.

2. Quasi-Creep Method

The internal pressure of the thick-walled vessel in each cycle increases from zero to a maximum value and then drops to zero. Therefore, the boundary conditions are:

\[
\begin{align*}
\sigma_{rr} &= 0 - P_i \quad r = R_i \\
\sigma_{rr} &= P_0 \quad r = R_0
\end{align*}
\] (1)

Ratcheting occurs when the equivalent method at each layer exceeds the yield limit. Therefore, the maximum pressure is selected so that the most outer vessel layer enters the plastic zone. Vessel temperature is considered as a constant and the same for the entire vessel. The following describes the variable material properties method and the image method.

2.1. Variable Material Properties Method

In this method, the vessel is divided into numerous layers with equal thicknesses. A small thickness is selected for each layer to assume that the mechanical properties along each layer are constant. Then the differential equation governing each layer is solved, and we can achieve an analytical-numerical solution for the intended problem by simultaneously solving the algebraic equations obtained from applying boundary conditions. By increasing the number of layers, the approximate solution will become closer to the exact solution.

2.2. The Image method

We can reach the elastoplastic solution from the pseudo-elastic solution by incorporating the image method and the variable material properties method. By defining Young's module and the equivalent Poisson coefficient [8], we can obtain the elastoplastic analysis similar to the elastic solution. The only difference with the elastic solution is that in this method, a pseudo-elastic solution first needs to be obtained with the equivalent values, and then the equivalent Young's module and the equivalent Poisson coefficient should be updated, and the solution must be repeated, so that the results match the stress-strain curve. In this method, the bilinear stress-strain curve is used.

With the assumption that the strain obtained from the pseudo-elastic method is smaller than that of the exact solution, Young's module is calculated using Equation 1. Figure 1 shows the schematic of the way the imaging is performed on the stress-strain curve, and that is why this method is called the image method. The equivalent stress is used to calculate the equivalent Poisson coefficient and the equivalent Young's module at each layer.

\[
E_{eff} = \frac{\sigma}{\varepsilon}|_{\alpha'}
\] (2)

2.3. Algorithm of the Quasi-Creep Analysis Method

Based on the descriptions above, the elastoplastic solution of the vessel can be found by repeatedly modifying the mechanical properties for each layer. With the help of the calculated stresses for each layer, the next repetitive loop begins with a specified number of repetitions to calculate the ratcheting strains. The variable material properties method is later used for the quasi-creep analysis of the ratcheting phenomenon. The results of this method are compared to those of finite element analysis in Abaqus Software.
2.4. Using the Thermoelastic Equation for the Governing Elastoplastic Analysis Using the Image Method

In order to reach the elastoplastic solution of the problem, the stress distribution in the vessel is obtained using the thermoelastic equation and the image method, which were explained before. The vessel equilibrium equations are:

\[
\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0
\]  
(3)

The relations between the stress and strain in the plane strain model are:

\[
\varepsilon_{rr} = \frac{1}{E} (\sigma_{rr} - \nu \sigma_{\theta\theta} - \nu \sigma_{zz}) + \alpha \Delta T
\]  
(4)

\[
\varepsilon_{\theta\theta} = \frac{1}{E} (\sigma_{\theta\theta} - \nu \sigma_{rr} - \nu \sigma_{zz}) + \alpha \Delta T
\]  
(5)

\[
\varepsilon_{zz} = 0
\]  
(6)

In these equations, \(\sigma_{rr}\) is the radial stress, \(\sigma_{zz}\) is the axial stress, and \(\sigma_{\theta\theta}\) is the hoop stress of the vessel. \(E\) is the elastic modulus, \(\nu\) is the Poisson coefficient, \(\alpha\) is the thermal expansion coefficient, and \(\Delta T\) is the vessel's temperature difference. These relations can be arranged based on the stresses:

\[
\sigma_{rr} = \frac{E}{(1 + \nu)(1 - 2\nu)} [(1 - \nu)\varepsilon_{rr} + \nu \varepsilon_{\theta\theta} - (1 + \nu)\alpha \Delta T]
\]  
(7)

\[
\sigma_{\theta\theta} = \frac{E}{(1 + \nu)(1 - 2\nu)} [(1 - \nu)\varepsilon_{\theta\theta} + \nu \varepsilon_{rr} - (1 + \nu)\alpha \Delta T]
\]  
(8)

Strain-displacement relations are shown in Equations 9 and 10.

\[
\varepsilon_{rr} = \frac{du}{dr}
\]  
(9)

\[
\varepsilon_{\theta\theta} = \frac{u}{r}
\]  
(10)

where \(u\), \(\varepsilon_{rr}\), and \(\varepsilon_{\theta\theta}\) are radial displacement, radial strain, and hoop strain, respectively. Using the given relations, the equilibrium equation governing the vessel can be converted to a differential equation based on displacement:

\[
(1 - \nu) \frac{d^2 u}{dr^2} \left[ \frac{\nu}{r} + \frac{1 - 2\nu}{r^2} \right] \frac{du}{dr} + \left[ -\frac{\nu}{r^2} - \frac{1 - 2\nu}{r^2} \right] u = 0
\]  
(11)

We can reach the solution by solving this differential equation using the numerical image method and by applying the boundary condition of the problem for each layer.
2.5. Quasi-Creep Governing Equations

The quasi-creep model presented in this research is based on the creep equation which governs the vessel. The way to obtain it is stated in the following. The quasi-creep analysis continues using the stresses obtained in the previous stage and based on the creep equations that govern the vessel. If we differentiate the vessel equilibrium equation, we have:

\[
\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0
\]  

(12)

The relation between stress and strain are:

\[
\dot{\epsilon}_{rr} = \dot{\epsilon}_{rr, T} + \dot{\epsilon}_{rr, c} = \frac{1}{E} \left( \sigma_{rr} - \nu \sigma_{\theta\theta} - \nu \sigma_{zz} \right) + \dot{\epsilon}_{rr, c}
\]  

(13)

\[
\varepsilon_{\theta\theta} = \dot{\epsilon}_{\theta\theta, T} + \dot{\epsilon}_{\theta\theta, c} = \frac{1}{E} \left( \sigma_{\theta\theta} - \nu \sigma_{rr} - \nu \sigma_{zz} \right) + \dot{\epsilon}_{\theta\theta, c}
\]  

(14)

where, \( \sigma_{rr} \) is the radial stress rate of the vessel, and \( \sigma_{\theta\theta} \) is the hoop stress rate of the vessel. Also, \( \dot{\epsilon}_{rr, c} \) is the radial creep strain rate, and \( \dot{\epsilon}_{\theta\theta, c} \) is the Hoop creep strain rate. Summing up Equations 13 and 14 with strain rates yields:

\[
\dot{\sigma}_{rr} = \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ (1 - \nu)\dot{\epsilon}_{rr} + \nu \dot{\epsilon}_{\theta\theta} - ((1 - \nu)\dot{\epsilon}_{rr, c} + \nu \dot{\epsilon}_{\theta\theta, c}) \right]
\]  

(15)

\[
\dot{\sigma}_{\theta\theta} = \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ (1 - \nu)\dot{\epsilon}_{\theta\theta} + \nu \dot{\epsilon}_{rr} - ((1 - \nu)\dot{\epsilon}_{\theta\theta, c} + \nu \dot{\epsilon}_{rr, c}) \right]
\]  

(16)

The strain rate- displacement rate relations in polar coordinates are shown in Equations 16 and 17.

\[
\dot{\epsilon}_{rr} = \frac{d\bar{u}}{dr}
\]  

(17)

\[
\dot{\epsilon}_{\theta\theta} = \frac{d\bar{u}}{r}
\]  

(18)

where, \( \bar{u}, \dot{\epsilon}_{rr}, \) and \( \dot{\epsilon}_{\theta\theta} \) are radial displacement rate, radial strain rate, and hoop strain rate, respectively. Norton-Bailey relations are used in the plane strain model to calculate the Hoop and radial strain rates.

\[
\dot{\epsilon}_{rr} = \frac{3kq\sigma_{eff}^{n-1}}{4} (\sigma_{rr} - \sigma_{\theta\theta}) N^{q-1}
\]  

(19)

\[
\dot{\epsilon}_{\theta\theta} = -\frac{3kq\sigma_{eff}^{n-1}}{4} (\sigma_{rr} - \sigma_{\theta\theta}) N^{q-1}
\]  

(20)

where N is the cycle, \( \sigma_{eff} \) is the equivalent von Mises stress and three parameters of \( \kappa, q \) and \( n \) are constant coefficients of the Norton-Bailey equation. The way to extract them will be stated in the following. Using the given relations, the equilibrium equation obtained based on the rate of stresses can be converted into a differential equation based on the displacement rate for each layer, which is shown in Equation 20.

\[
(1 - \nu) \frac{d^2 \bar{u}}{dr^2} + \frac{(1 - \nu)}{r} \frac{d\bar{u}}{dr} + \frac{(\nu - 1)}{r^2} \bar{u} + \frac{(2 \nu - 1)}{2} \dot{\epsilon}_{rr} - \frac{(2 \nu - 1)}{r} \dot{\epsilon}_{rr} - (\nu - 1) \frac{d\dot{\epsilon}_{\theta\theta}}{dr} - \nu \frac{d\dot{\epsilon}_{rr, c}}{dr} = 0
\]  

(21)

The quasi-creep strains will be obtained by solving this differential equation for each layer and applying the boundary condition.

3. Extracting the Constants of the Norton-Bailey Equation

We use the finite element analysis of the ratcheting behavior of a rod using the Chaboche model to extract the constants of the Norton-Bailey equation in the quasi-creep analysis. For this purpose, a rod is subjected to cycling loadings several times with different loads and eventually, we use the obtained results to extract the fitness and constants, which are shown in Table 1.

4. Validation

The finite element solution in Abaqus Software is used to validate the quasi-creep method. The Chaboche model is used with three back stresses to apply the maximum accuracy. The used constants of Reference [10] in finite element analysis of the ratcheting behaviour are shown in Table 2.
Table 1. Fixed constants in the quasi-creep equation

<table>
<thead>
<tr>
<th>Column</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\kappa$</td>
<td>9.06e-56</td>
</tr>
<tr>
<td>2</td>
<td>$q$</td>
<td>0.13−8$\sigma_{f}$+0.937</td>
</tr>
<tr>
<td>3</td>
<td>$n$</td>
<td>6.16</td>
</tr>
</tbody>
</table>

Table 2. Necessary constants for finite element analysis based on the Chaboche model in Abaqus Software

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>198Gpa</td>
<td>$C_3$</td>
<td>4264 Mpa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>$\gamma_1$</td>
<td>4407</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>137.45</td>
<td>$\gamma_2$</td>
<td>366</td>
</tr>
<tr>
<td>$C_1$</td>
<td>16137 Mpa</td>
<td>$\gamma_3$</td>
<td>9</td>
</tr>
<tr>
<td>$C_2$</td>
<td>71250 Mpa</td>
<td>Q</td>
<td>9924 Mpa</td>
</tr>
<tr>
<td>$b$</td>
<td>0.0135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Results

In order to validate the quasi-creep method, we perform calculations for a vessel with inner and outer radiuses of 0.2m and 0.3m, respectively and a maximum internal pressure of 200MPa and a constant external pressure of 100KPa for 50 cycles. The results of both the quasi-creep method and finite element analysis were extracted based on the Chaboche model.

Figure 2 shows the hoop strains for three inner, middle, and outer layers during 50 cycles for both methods. Abaqus results show strain changes in each cycle, but only the maximum strain in each cycle is calculated in the quasi-creep method. The same results were extracted for radial strains and are shown in Figure 3, and the accuracy of results are acceptable like the results of hoop strains.

The final Hoop and radial strains along the radius were extracted after 50 cycles and are shown in Figures 4 and 5. There is a good correlation between the results of both methods, which implies the high efficiency of the quasi-creep method. It is also seen that the amplitudes of strains are at their maximum in the most inner layer and reduces to the most outer layer.

The drawn graphs clearly show how good the quasi-creep method can predict ratcheting strains in a thick-walled vessel. For more clarification on the quasi-creep method, the convergence graph of the pseudo-elastic solution into the elastoplastic solution is shown in Figure 6.

The stress-strain curve of the most inner layer of the vessel was also extracted from the Abaqus Software based on the combined non-linear hardening model for 25 cycles and is shown in Figure 7.

![Figure 2. Comparing the hoop strains in respect to cycles](image)
Figure 3. Comparing radial strains in respect to cycles

Figure 4. Comparing hoop strains after 50 cycles

Figure 5. Comparing the radial strains after 50 cycles

Figure 6. Convergence of the pseudo-tensile analysis into elastoplastic analysis
As it can be seen, the quasi-creep method can predict ratcheting strains with a desirable accuracy, and in addition to this, the exceptionally high speed of this method is its advantage against the finite element analysis. Computing the ratcheting strains for 50 cycles in the quasi-creep method takes less than 5 seconds, but analyzing the same vessel using finite element analysis requires more than 400 seconds.

Figures 8 and 9 show the Hoop and radial ratcheting strains in the most inner layer of the vessel for internal pressures of 180, 190, 200, and 210MPA, respectively. Comparing the results show that a 5% increase in the maximum internal pressure significantly increases the ratcheting strains. Figures 10 and 11 show the Hoop and radial ratcheting strain for different thicknesses of the most inner layer of the vessel. Comparing the results demonstrates that a 10% increase in thinkness will reduce the strains by 15%.

![Figure 7. The hoop stress-hoop strain curve for 25 cycles in the combined hardening model in the Abaqus Software](image)

Figure 8. Comparing the hoop strain for different internal pressures

![Figure 9. Comparing the radial strain for different internal pressures](image)
6. Conclusions

This study presented a method to predict the ratcheting behaviour in thick-walled vessels under compressive cyclic loading. Due to many similarities between the ratcheting and creep phenomena, a quasi-creep method was presented. This method combines the variable material properties method, pseudo-elastic solution, the image method and the creep analysis. Finite element analysis was performed based on the Chaboche model for validation, and the results of these two methods were compared, which showed the accuracy and high speed of the quasi-creep method. Using this method, the effect of the thick-walled vessel’s maximum internal pressure and thickness on ratcheting strains was examined. The results showed that ratcheting strains drop because of reduction of maximum internal pressure and increase in thickness. Other parameters such as the ratio of radiuses, vessel temperature and mechanical features can be assessed using this method.

References


